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VIBRATION CONTROL OF SHALLOW SHELL STRUCTURES USING A SHELL-TYPE DYNAMIC VIBRATION ABSORBER

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In this study, a new shell-type dynamic vibration absorber is presented for suppressing several modes of vibration of the shallow shell (main shell) under harmonic load. It consists of a shallow shell (the dynamic absorbing shell), under the same boundary condition and with the same shape as those of the main shell, with connecting springs and dampers in the vertical direction between the main and dynamic absorbing shells. Formulae for an approximate tuning method for the shell-type dynamic absorber are also presented using the optimum tuning method for a dynamic absorber in the two-degree-of-freedom system, obtained by the Den Hartog method. Subsequently, numerical calculations are presented which demonstrate the usefulness of the shell-type dynamic vibration absorbers.

1. INTRODUCTION

Dynamic vibration absorbers with high control performance, which can be used to suppress several vibration modes, are required in practical usage. Many ways of contriving the dynamic vibration absorbers have been considered. However, one of the most reliable devices for passive control of structures under harmonic excitation is the tuned mass damper (TMD). The device comprises a mass, a spring and a damper. TMDs can also be used effectively for controlling the bending vibration of shell structures [1, 2]. In this paper, a new shell-type dynamic vibration

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absorbing system is proposed. It could be used to control several vibration modes of the shallow shell (main shell), which can be fitted to the equivalent isotropic or orthotropic shallow shell outside plate structures with a slight curvature, such as vessels, ships, roofs of buildings and so on. This shell-type dynamic absorber consists of a shallow shell (dynamic absorbing shell), under the same boundary conditions and with the same shape as those of the main shell, with uniformly distributed connecting springs and dampers in the vertical and two horizontal directions of the shell. The springs and dampers are situated between the main and dynamic absorbing shells, as shown in Figure 1. A structural member, with the shell-type dynamic absorber mentioned above, is like a constrained layered sandwich plate, which is curved in appearance, but basically different because mechanical springs and dampers are used instead of the constraining viscoelastic layer of the sandwich plates. For practical use, concentrated connecting springs and dampers in the vertical direction are used in this new dynamic absorber. This shell-type dynamic absorber will be useful for some of the above-mentioned structure parts.

The authors have proposed new beam-type [3] and plate-type dynamic vibration absorbers [4] to control the bending vibration of a beam (main beam) and a plate (main plate). Each consists of a beam (dynamic absorbing beam) and a plate (dynamic absorbing plate), under the same boundary conditions as those of the main beam and plate, respectively, with uniformly distributed connecting springs

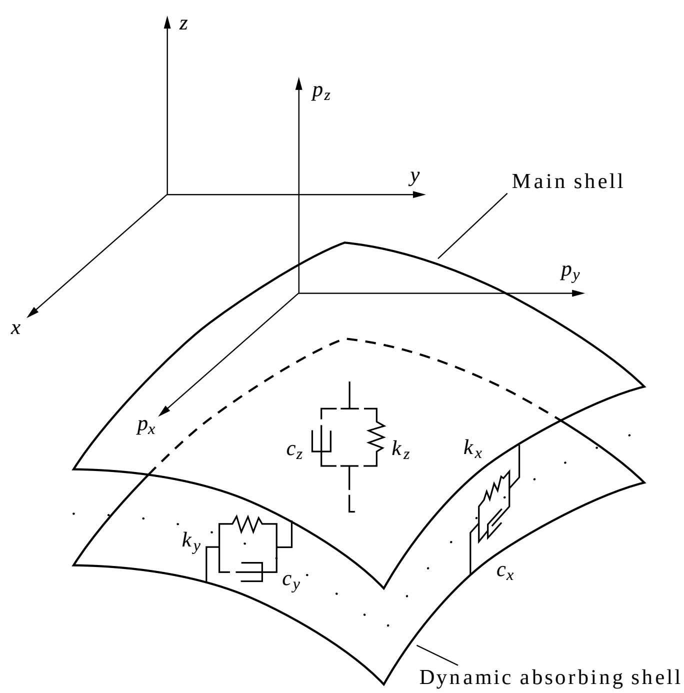

Figure 1. Shell with the dynamic absorbing shell attached by the uniformly distributed springs and dampers.

and dampers. They are situated between the main and dynamic absorbing beams and between the main and dynamic absorbing plates, respectively. Tuning methods for the new dynamic absorbers mentioned above were also developed. Several sets of concentrated connecting springs and dampers arranged at regular intervals on the dynamic absorbing beam and plate are more practical, and can be used instead of uniformly distributed ones. In this paper, a tuning method for the shell-type dynamic absorber with uniformly distributed connecting springs and dampers is presented using the same procedure as those of the beam-type and plate-type dynamic absorbers mentioned previously. For its practicality, an approximate tuning method for the shell-type dynamic absorber with several sets of concentrated connecting springs and dampers in the vertical direction is also proposed. Because the dynamic displacements in the two horizontal directions of the shallow shell are thought to be far smaller than the vertical displacement, vertical connecting springs and dampers are used exclusively in suppressing the vibration of the shallow shell. Moreover, to make large-scale problems manageable, concentrated connecting springs and dampers, which are arranged at regular intervals in the two horizontal directions, are used.

First, the authors consider two systems consisting of main and dynamic absorbing shells; one has the uniformly distributed connecting springs and dampers and the other has concentrated ones in the vertical direction. The equations of motion of the two systems mentioned above, in the modal co-ordinates of the main shell, are found to be equivalent to those of a system with two masses and three springs. Tuning methods for the two shell-type dynamic vibration absorbers are presented, based on the optimum tuning method of the dynamic vibration absorber in the two-degree-of-freedom system, which was obtained by the Den Hartog method [5]. Finally, the usefulness of the new dynamic vibration absorber proposed here and the applicability of an approximate tuning method are shown using numerical examples. The influence of the arrangement of the concentrated connecting springs and dampers on the suppressing effect is then investigated numerically.

2. EQUATIONS OF MOTION OF SYSTEMS AND THEIR MODAL EQUATIONS

The two shallow shells shown in Figures 1 and 2 were chosen as the main shells to which the dynamic absorbing shells are attached by connecting springs and dampers, under the same boundary condition, and with the same shape as those of the main shells. Figure 1 shows the main shell with the shell-type dynamic absorber with uniformly distributed connecting springs and dampers in the vertical and two horizontal directions. Figure 2 shows the main shell with a dynamic absorbing shell attached by $(J \times K)$ sets of concentrated connecting springs and dampers in the vertical direction. They are arranged in a rectangular lattice shape with proper separations, as shown in Figure 3. Equations of motion and modal

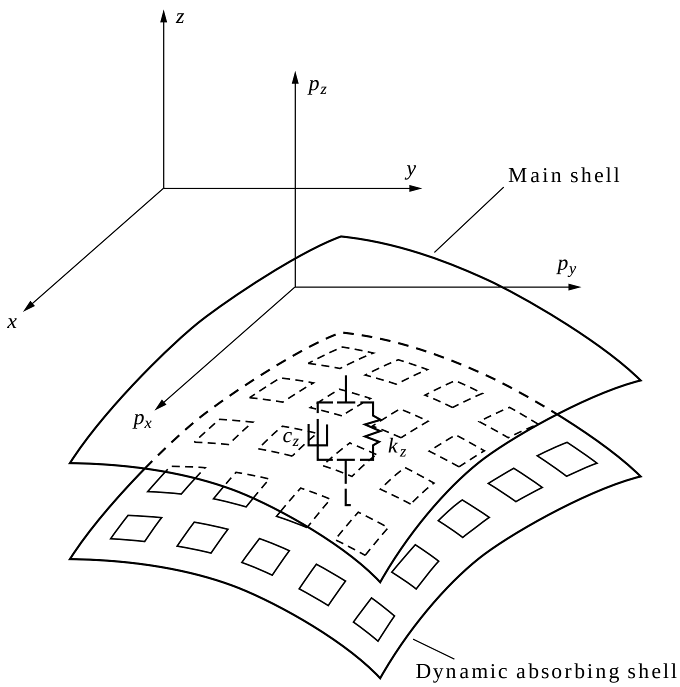

Figure 2. Shell with the dynamic absorbing shell attached by the concentrated springs and dampers in the vertical direction.

equations were derived based on the following assumptions: (1) displacements of the shell are small in comparison with its thickness, which is small compared to the other dimensions of the shell and its radii of curvature; (2) curvatures and rate of twist of the shell surface are very small, i.e., the shells are platelike shallow ones; (3) shear strain in each shell as well as the rotatory inertia of a cross-section are neglected; (4) structural internal damping of a shell is proportional to the velocity

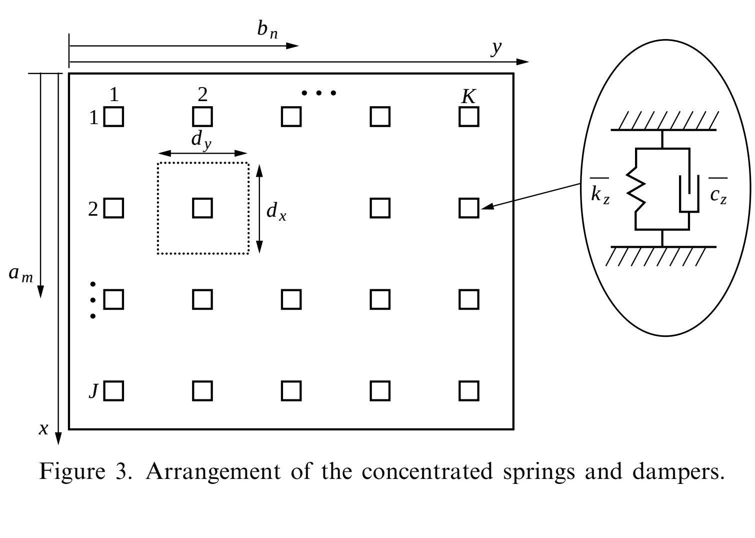

Figure 3. Arrangement of the concentrated springs and dampers.

of displacement; (5) connecting springs and dampers are effective in the direction of their axes, but not in the other directions; (6) the masses of the connecting springs and dampers are neglected; and (7) eigenfunctions of the shells without a dynamic vibration absorber are used as the mode shape function.

In deriving the equations of motion, the co-ordinates and load conditions shown in Figures 1 and 2 are used. Equations of motion of the main and dynamic absorbing shells which are platelike and shallow, with uniform flexural and extensional rigidities, and uniform mass per unit area, are expressed as follows:

main shell

$$m_1\{\ddot{q}_1\} - K_1[S_1]\{q_1\} + [c](\{\dot{q}_1\} - \{\dot{q}_2\}) + [k](\{q_1\} - \{q_2\}) = \{P\}\delta(x-r)\delta(y-s)\,\mathrm{e}^{i\omega_0 t}, \tag{1}$$

dynamic absorbing shell

$$m_2\{\ddot{q}_2\} - K_2[S_2]\{q_2\} + [c](\{\dot{q}_2\} - \{\dot{q}_1\}) + [k](\{q_2\} - \{q_1\}) = \{0\}, \tag{2}$$

in which

$$\{q_1\} = \{u_1, v_1, w_1\}^T, \qquad \{q_2\} = \{u_2, v_2, w_2\}^T, \tag{3}$$

$$[S_1] = \begin{bmatrix} \dfrac{\partial^2}{\partial x^2} + \dfrac{1-v}{2}\dfrac{\partial^2}{\partial y^2} & \dfrac{1+v}{2}\dfrac{\partial^2}{\partial x\,\partial y} & -\left(L_x\dfrac{\partial}{\partial x} + L_{xy}\dfrac{\partial}{\partial y}\right) \\ \dfrac{1+v}{2}\dfrac{\partial^2}{\partial x\,\partial y} & \dfrac{\partial^2}{\partial y^2} + \dfrac{1-v}{2}\dfrac{\partial^2}{\partial x^2} & -\left(L_y\dfrac{\partial}{\partial y} + L_{xy}\dfrac{\partial}{\partial x}\right) \\ \left(L_x\dfrac{\partial}{\partial x} + L_{xy}\dfrac{\partial}{\partial y}\right) & \left(L_y\dfrac{\partial}{\partial y} + L_{xy}\dfrac{\partial}{\partial x}\right) & -L_0 - \dfrac{D_1}{K_1}\left(\dfrac{\partial^4}{\partial x^4} + 2\dfrac{\partial^4}{\partial x^2\,\partial y^2} + \dfrac{\partial^4}{\partial y^4}\right) \end{bmatrix}, \tag{4}$$

$$[S_2] = \begin{bmatrix} \dfrac{\partial^2}{\partial x^2} + \dfrac{1-v}{2}\dfrac{\partial^2}{\partial y^2} & \dfrac{1+v}{2}\dfrac{\partial^2}{\partial x\,\partial y} & -\left(L_x\dfrac{\partial}{\partial x} + L_{xy}\dfrac{\partial}{\partial y}\right) \\ \dfrac{1+v}{2}\dfrac{\partial^2}{\partial x\,\partial y} & \dfrac{\partial^2}{\partial y^2} + \dfrac{1-v}{2}\dfrac{\partial^2}{\partial x^2} & -\left(L_y\dfrac{\partial}{\partial y} + L_{xy}\dfrac{\partial}{\partial x}\right) \\ \left(L_x\dfrac{\partial}{\partial x} + L_{xy}\dfrac{\partial}{\partial y}\right) & \left(L_y\dfrac{\partial}{\partial y} + L_{xy}\dfrac{\partial}{\partial x}\right) & -L_0 - \dfrac{D_2}{K_2}\left(\dfrac{\partial^4}{\partial x^4} + 2\dfrac{\partial^4}{\partial x^2\,\partial y^2} + \dfrac{\partial^4}{\partial y^4}\right) \end{bmatrix}, \tag{5}$$

$$[c] = \begin{cases} \begin{bmatrix} c_x & & \\ & c_y & \\ & & c_z \end{bmatrix} & \text{for uniformly distributed connecting dampers} \\ \begin{bmatrix} 0 & & \\ & 0 & \\ & & \bar{c}_z \sum_{m=1}^{J} \sum_{n=1}^{K} \delta(x - a_m)\delta(y - b_n) \end{bmatrix} & \text{for concentrated connecting dampers} \end{cases}, \quad (6)$$

$$[k] = \begin{cases} \begin{bmatrix} k_x & & \\ & k_y & \\ & & k_z \end{bmatrix} & \text{for uniformly distributed connecting springs} \\ \begin{bmatrix} 0 & & \\ & 0 & \\ & & \bar{k}_z \sum_{m=1}^{J} \sum_{n=1}^{K} \delta(x - a_m)\delta(y - b_n) \end{bmatrix} & \text{for concentrated connecting springs} \end{cases}, \quad (7)$$

$$\{P\} = \{P_x, P_y, P_z\}^T, \tag{8}$$

$$L_x = \rho_x + v\rho_y, \qquad L_y = \rho_y + v\rho_x, \qquad L_{xy} = (1 - v)\rho_{xy},$$
$$L_0 = \rho_x^2 + 2(1 - v)\rho_{xy}^2 + \rho_y^2 + 2v\rho_x\rho_y. \tag{9}$$

In the above expressions, u_1, v_1 and w_1 are displacements in the x, y and z directions of the main shell; u_2, v_2 and w_2 are displacements of the dynamic absorbing shell; m_1 and m_2 are masses per unit area of both shells above; D_1 and D_2 are flexural rigidities of both shells; K_1 and K_2 are extensional rigidities of both shells; v is the Poisson ratio of the material of both shells; k_x, k_y and k_z are spring constants for uniformly distributed connecting springs in the x, y and z directions; c_x, c_y and c_z are damping coefficients of the uniformly distributed connecting dampers in the x, y and z directions; $\overline{k}_z$ and $\bar{c}_z$ are respectively the spring constant of concentrated connecting springs in the vertical direction, and the damping coefficient of the concentrated connecting damper in the vertical direction; ρ_x and ρ_y are curvatures of the shell surface with respect to the x and y axes, and ρ_{xy} is a rate of twist for the shell surface; P_x, P_y and P_z are the amplitudes of the external loads in the x, y and z directions; ω_0 is the exciting frequency; δ is the Dirac delta function; (r, s) are co-ordinates of the load position; (a_m, b_n) $(m = 1, 2, \ldots, J, n = 1, 2, \ldots, K)$ are position co-ordinates of the concentrated connecting springs and dampers.

The eigenfunctions of the main and dynamic absorbing shells are expressed by the same functions when both shells have the same shape and boundary conditions, and when the following relations with respect to the mass and rigidities of both shells hold:

$$\frac{m_2}{m_1} = \alpha, \qquad \frac{K_2}{K_1} = \frac{D_2}{D_1} = \beta, \tag{10}$$

in which α and β are constant.

The normalized eigenfunctions of the ith mode of both shells are denoted by $U_i(x, y)$, $V_i(x, y)$ and $W_i(x, y)$. The orthogonal condition of normal modes is given as

$$\int_0^a \int_0^b \{Q_i\}^T\{Q_j\}\,\mathrm{d}x\,\mathrm{d}y = \begin{cases} \gamma \cdots i = j \\ 0 \cdots i \neq j \end{cases}, \tag{11}$$

in which

$$\{Q_i\} = \{U_i, V_i, W_i\}^T, \qquad \{Q_j\} = \{U_j, V_j, W_j\}^T. \tag{12}$$

When the natural circular frequencies of the ith mode of both shells are denoted by ω_{1i} and ω_{2i}, respectively, these frequencies and the eigenfunctions satisfy the following relations:

$$K_1[S_1]\{Q_i\} = m_1\omega_{1i}^2\{Q_i\} \tag{13}$$

for the main shell and

$$K_2[S_2]\{Q_i\} = m_2\omega_{2i}^2\{Q_i\} \tag{14}$$

for the dynamic absorbing shell. Accordingly, the following relation between the natural circular frequencies of both shells, ω_{1i} and ω_{2i}, holds:

$$\frac{m_1\omega_{1i}^2}{K_1} = \frac{m_2\omega_{2i}^2}{K_2}. \tag{15}$$

The following approximate functions can be used for solutions of equations (1) and (2):

$$\{q_1\} = \sum_{i=1}^{M} \{Q_i\}\rho_{1i}(t) \tag{16}$$

for the main shell and

$$\{q_2\} = \sum_{i=1}^{M} \{Q_i\}\rho_{2i}(t) \tag{17}$$

for the dynamic absorbing shell, in which $\rho_{1i}(t)$ and $\rho_{2i}(t)$ are unknown functions of the time of the main and dynamic absorbing shells.

When damping coefficients and spring constants in three directions are equal to each other as:

$$c_x = c_y = c_z = c \quad \text{and} \quad k_x = k_y = k_z = k, \tag{18}$$

substitution of equations (16) and (17) into equations (1) and (2) and their rearrangement according to equations (11)–(15) give the following modal equations for the shell-type absorber with uniformly distributed connecting springs and dampers:

$$m_1\ddot{\rho}_{1j} + m_1\omega_{1j}^2\rho_{1j} + c(\dot{\rho}_{1j} - \dot{\rho}_{2j}) + k(\rho_{1j} - \rho_{2j}) = \frac{1}{\gamma}\,(P_x U_j(r,s) + P_y V_j(r,s) + P_z W_j(r,s))\, \mathrm{e}^{i\omega_0 t}, \tag{19}$$

$$m_2\ddot{\rho}_{2j} + m_2\omega_{2j}^2\rho_{2j} + c(\dot{\rho}_{2j} - \dot{\rho}_{1j}) + k(\dot{\rho}_{2j} - \rho_{1j}) = 0, \qquad (j = 1, 2, \ldots, M). \tag{20}$$

In the case of the shell-type absorber with concentrated connecting springs and dampers in the vertical direction, the modal equations become:

$$m_1\ddot{\rho}_{1j} + m_1\omega_{1j}^2\rho_{1j} + \frac{\bar{c}_z}{\gamma}\sum_{i=1}^{M}\kappa_{ij}(\dot{\rho}_{1i} - \dot{\rho}_{2i}) + \frac{\overline{k}_z}{\gamma}\sum_{i=1}^{M}\kappa_{ij}(\rho_{1i} - \rho_{2i}) = \frac{1}{\gamma}\,\{P_x U_j(r,s) + P_y V_j(r,s) + P_z W_j(r,s)\}\, \mathrm{e}^{i\omega_0 t}, \tag{21}$$

$$m_2\ddot{\rho}_{2j} + m_2\omega_{2j}^2\rho_{2j} + \frac{\bar{c}_z}{\gamma}\sum_{i=1}^{M}\kappa_{ij}(\dot{\rho}_{2i} - \dot{\rho}_{1i}) + \frac{\overline{k}_z}{\gamma}\sum_{i=1}^{M}\kappa_{ij}(\rho_{2i} - \rho_{1i}) = 0, \quad (j = 1, 2, \ldots, M), \tag{22}$$

in which

$$\kappa_{ij} = \sum_{m=1}^{J} \sum_{n=1}^{K} W_i(a_m, b_n) W_j(a_m, b_n). \tag{23}$$

Let the spring constant $\bar{k}_z$ and the damping coefficient $\bar{c}_z$ be expressed as follows:

$$\bar{k}_z = (d_x \cdot d_y)\tilde{k}, \qquad \bar{c}_z = (d_x \cdot d_y)\tilde{c}, \tag{24}$$

where $\tilde{k}$ and $\tilde{c}$ are the equivalent spring constant and damping coefficient of the connecting spring and damper distributed uniformly, and d_x and d_y are the distances in the x and y directions between the sets of concentrated connecting springs and dampers. The second and third terms of the left side of equations (21) and (22) can be expressed as:

$$\bar{c}_z \sum_{i=1}^{M} \kappa_{ij}(\dot{\rho}_{1i} - \dot{\rho}_{2i}) = \tilde{c} \sum_{i=1}^{M} \Phi_{ij}(\dot{\rho}_{1i} - \dot{\rho}_{2i}),$$

$$\bar{k}_z \sum_{i=1}^{M} \kappa_{ij}(\rho_{1i} - \rho_{2i}) = \tilde{k} \sum_{i=1}^{M} \Phi_{ij}(\rho_{1i} - \rho_{2i}), \tag{25}$$

where

$$\Phi_{ij} = \sum_{m=1}^{J} \sum_{n=1}^{K} W_i(a_m, b_n) W_j(a_m, b_n) d_x d_y. \tag{26}$$

When the shells are platelike shallow ones, and intervals d_x and d_y are small, the following relation should hold:

$$\Phi_{ij} \approx \int_0^a \int_0^b W_i(x, y) W_j(x, y)\, \mathrm{d}x\, \mathrm{d}y \approx \int_0^a \int_0^b \{U_i, V_i, W_i\}\{U_j, V_j, W_j\}^T\, \mathrm{d}x\, \mathrm{d}y. \tag{27}$$

The right side of the above equation is approximately equal to the orthogonal condition of the modes of bending vibration for a shell having equal mass per unit area. This is because eigenfunctions U_j and V_j $(j = 1, 2, \ldots, M)$ are so much smaller than W_j in the free vibration of a platelike shallow shell. Accordingly, Φ_{ij} has the following values:

$$\Phi_{ij} \approx \begin{cases} \gamma \cdots i = j \\ 0 \cdots i \neq j \end{cases}. \tag{28}$$

Equations (21) and (22) can be rewritten approximately as follows:

$$m_1\ddot{\rho}_{1j} + m_1\omega_{1j}^2\rho_{1j} + \tilde{c}(\dot{\rho}_{1j} - \dot{\rho}_{2j}) + \tilde{k}(\rho_{1j} - \rho_{2j})$$

$$= \frac{1}{\gamma}\{P_x U_j(r, s) + P_y V_j(r, s) + P_z W_j(r, s)\}\, e^{i\omega_0 t}, \tag{29}$$

$$m_2\ddot{\rho}_{2j} + m_2\omega_{2j}^2\rho_{2j} + \tilde{c}(\dot{\rho}_{2j} - \dot{\rho}_{1j}) + \tilde{k}(\rho_{2j} - \rho_{1j}) = 0 \quad (j = 1, 2, \ldots, M). \tag{30}$$

Modal equations (29) and (30) also have the same form as modal equations (19) and (20). These modal equations correspond to the equations of motion of the two-degree-of-freedom system (TDOF system) shown in Figure 4, substituting as follows:

$$M_1 = m_1, \qquad M_2 = m_2, \qquad k_1 = m_1\omega_{1j}^2, \qquad k_2 = k, \qquad c_2 = c, \qquad k_3 = m_2\omega_{2j}^2,$$

$$P_0 = \frac{1}{\gamma}\{P_x U_j(r, s) + P_y V_j(r, s) + P_z W_j(r, s)\} \tag{31}$$

for the shell-type dynamic absorber with uniformly distributed connecting springs and dampers and

$$M_1 = m_1, \qquad M_2 = m_2, \qquad k_1 = m_1\omega_{1j}^2, \qquad k_2 = \tilde{k} = \frac{\bar{k}_z}{d_x d_y},$$

$$c_2 = \tilde{c} = \frac{\bar{c}_z}{d_x d_y}, \qquad k_3 = m_2\omega_{2j}^2,$$

$$P_0 = \frac{1}{\gamma}\{P_x U_j(r, s) + P_y V_j(r, s) + P_z W_j(r, s)\} \tag{32}$$

for the shell-type dynamic absorber with concentrated connecting springs and dampers.

3. TUNING METHOD OF SHELL-TYPE DYNAMIC VIBRATION ABSORBERS

Displacement of the main shell during vibration from the periodic load can be minimized by controlling the unknown function of time, $\rho_{1i}(t)$, in equation (16). Suppose that the exciting frequency of load, ω_0, is close to the natural circular frequency of the jth mode of the main shell, ω_{1j}, which is not very close to the other. In this forced vibration, the jth mode is predominant and displacements of the main shell are given approximately by equation (16) as the following expression:

$$\{q_1\} \approx \{Q_j\}\rho_{1j}(t). \tag{33}$$

Therefore, displacements of the main shell, $\{q_1\}$, can be approximately minimized by control of the normal co-ordinate of the jth mode, ρ_{1j}.

The behavior of ρ_{1j} is described by the vibration of the main system in the TDOF system shown in Figure 4. The optimal tuning conditions of the dynamic vibration absorber (subsystem) in the TDOF system are given by the Den Hartog method as follows [3].

(1) The first tuning condition, under which the ordinates of the two fixed points through which every resonance curve of the main system for varying c_2 must pass are equal, is given as:

$$\frac{k_2}{\mu k_1} = \frac{1}{(1+\mu)^2}\left(1 - \frac{k_3}{\mu k_1}\right), \tag{34}$$

in which

$$\mu = \frac{M_2}{M_1}. \tag{35}$$

In this case, the ordinates of the fixed point, Y_{1p}, are

$$Y_{1p} = \frac{1}{(1 - k_3/\mu k_1)}\sqrt{\frac{2+\mu}{\mu}}, \tag{36}$$

where Y_{1p} is the non-dimensional amplitude of the main system (magnification factor) and is denoted by

$$Y_{1p} = \frac{\mathrm{Re}\,(\rho_{1j})}{y_{st}} \geqslant \sqrt{\frac{2+\mu}{\mu}}, \tag{37}$$

in which $\mathrm{Re}\,(\rho_{1j})$ is the real amplitude of ρ_{1j} and y_{st} is the static displacement to be given as:

$$y_{st} = \frac{P_0}{k_1}. \tag{38}$$

(2) The second tuning condition, under which the maximum point of the resonance curve is the point mentioned above, is satisfied when the damping coefficient c_2 is given as follows:

$$c_2 = 2\mu h\sqrt{k_1 m_1} \tag{39}$$

in which

$$h^2 = \tfrac{1}{2}(h_P^2 + h_Q^2) \tag{40}$$

and

$$\left.\begin{matrix} h_P^2 \\ h_Q^2 \end{matrix}\right\} = \frac{(3+2\mu) \mp 2\sqrt{\mu(2+\mu)}}{4(1+\mu)^3 Y_{1p}\{(1+\mu)Y_{1p} - \sqrt{\mu(2+\mu)} \mp 1\}}. \tag{41}$$

The dynamic vibration absorber in the TDOF system with two masses and three springs is designed optimally by the following procedures: (1) the mass of the dynamic vibration absorber, $M_2(\mu)$, and the amplitude limit of the main system, Y_{1p}, are assumed; (2) the spring constant of the dynamic vibration absorber, k_3, for μ and Y_{1p} assumed previously, is estimated by equation (36); (3) the spring constant of the connecting spring, k_2, for μ and k_3 is estimated by equation (34); and (4) the damping coefficient c_2 of the connecting damper is estimated by equations (39)–(41).

The tuning method for the shell-type dynamic absorber for control of the jth mode of the main shell is given by control of ρ_{1j}, using the tuning conditions of the dynamic absorber in the TDOF system mentioned previously, and is shown hereafter.

(1) The ratio of the mass of the main shell to that of the dynamic absorbing shell, $\alpha = m_2/m_1$, and the limit amplitude at a given point (x_0, y_0) of the main shell, δ_{max}, are set before beginning the calculation. In this case, the mass ratio $\mu = M_2/M_1$ and the static displacement of the main system, y_{st}, in the TDOF system, shown in Figure 4, are expressed from equations (31), (32), (35) and (38) as:

$$\mu = \frac{M_2}{M_1} = \frac{m_2}{m_1} = \alpha \tag{42}$$

and

$$y_{st} = \frac{P_0}{k_1} = \frac{1}{\gamma m_1 \omega_{1j}^2}\left\{P_x U_j(r, s) + P_y V_j(r, s) + P_z W_j(r, s)\right\}. \tag{43}$$

Displacement of a given point on the main shell in a vibrating state, $\delta(x_0, y_0, t)$, is denoted approximately from equations (33) and (37) as:

$$\delta(x_0, y_0, t) \approx \Delta_j(x_0, y_0)\rho_{1j} \leqslant \Delta_j(x_0, y_0)\,\mathrm{Re}\,(\rho_{1j}) = \Delta_j(x_0, y_0) Y_{1p} y_{st} \leqslant \delta_{max}, \tag{44}$$

in which

$$\Delta_j(x_0, y_0) = \sqrt{U_j(x_0, y_0)^2 + V_j(x_0, y_0)^2 + W_j(x_0, y_0)^2}. \tag{45}$$

Accordingly, the non-dimensional amplitude of the main system in the TDOF system, Y_{1p}, is given as follows:

$$\frac{\delta_{max}}{\Delta_j(x_0, y_0) y_{st}} = \frac{\gamma m_1 \omega_{1j}^2 \delta_{max}}{(P_x U_j(r, s) + P_y V_j(r, s) + P_z W_j(r, s))\Delta_j(x_0, y_0)} \geqslant Y_{1p} \geqslant \sqrt{\frac{2+\mu}{\mu}}. \tag{46}$$

(2) When μ and Y_{1p} in the TDOF system are obtained by equations (42) and (46), the flexural and extensional rigidities of the dynamic absorbing shell, D_2 and K_2, are given from equations (31), (32), (15) and (36) as:

$$D_2 = D_1 \mu\left(1 - \frac{1}{Y_{1p}}\sqrt{\frac{2+\mu}{2}}\right) \tag{47}$$

and

$$K_2 = K_1 \mu\left(1 - \frac{1}{Y_{1p}}\sqrt{\frac{2+\mu}{2}}\right). \tag{48}$$

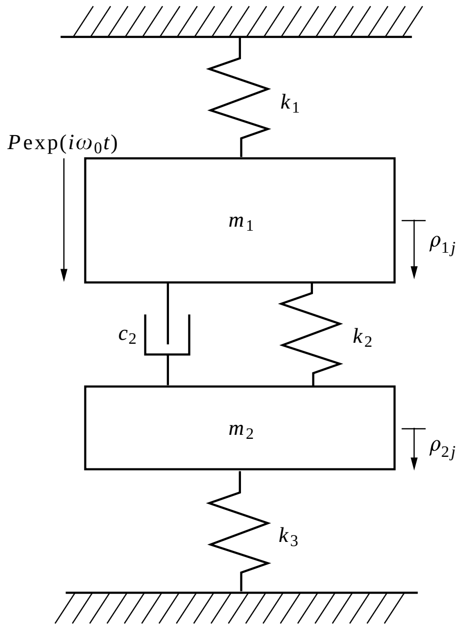

Figure 4. Two-degree-of-freedom system for the *j*th mode of a main shell with shell-type dynamic absorber.

(3) The spring constants of the connecting springs, k and $\overline{k}_z$, are given by equations (31), (32) and (34) as:

$$k = \frac{m_1 \omega_{1j}^2 \sqrt{\mu(2+\mu)}}{Y_{1p}(1+\mu)^2} \tag{49}$$

for the uniformly distributed connecting springs and

$$\overline{k}_z = \frac{m_1 \omega_{1j}^2 \sqrt{\mu(2+\mu)}}{Y_{1p}(1+\mu)^2} d_x d_y \tag{50}$$

for the concentrated connecting springs.

(4) Damping coefficients of the connecting dampers, c and $\bar{c}_z$, are given from equations (31), (32) and (39) as:

$$c = 2\mu h m_1 \omega_{1j} \tag{51}$$

for the uniformly distributed connecting dampers and

$$\bar{c}_z = 2\mu h m_1 \omega_{1j} d_x d_y \tag{52}$$

for the concentrated connecting dampers, in which h is estimated by equations (40) and (41).

4. NUMERICAL INVESTIGATIONS

Two platelike shallow cylindrical shells with the geometrical and structural constants shown in Table 1, one supported simply at the circumference and the other fixed, were chosen for numerical investigations. Dynamic characteristics of the shells are illustrated in Figures 5 and 6. In numerical investigations, the shell-type dynamic absorber with uniformly distributed connecting springs and dampers in the vertical and two horizontal directions of the shell, and the shell-type dynamic absorber with several sets of concentrated connecting springs and dampers in the vertical direction, were considered and were tuned using the optimum tuning method presented here. The usefulness of the shell-type dynamic absorbers and of the tuning methods presented here was demonstrated by illustrating the resonance curves of the main shells with the above shell-type dynamic absorbers. Moreover, changes in the suppressing effect on the vibration, due to variation in the arrangement of concentrated connecting springs and dampers, were investigated.

The resonance curves of the main shells, illustrated from the following numerical investigations, are dynamic responses of the vertical displacement at a loop point of the 1st mode of the main shell under a harmonic load acting on the same point. The loading states of the main shells are shown in Figures 5 and 6. The resonance curves of the dynamic absorbing shells are dynamic responses of the vertical displacement at a point right under the loading point of the main shell.

In calculating the dynamic responses, the displacements of the main shell with the uniformly distributed connecting springs and dampers were obtained from the partial sum of the 30 terms in equation (16) with function ρ_{1j} obtained from equations (19) and (20). The displacements of the main shell with the concentrated connecting springs and dampers were calculated from the partial sum of the 30 terms in equation (16) with function ρ_{1j} obtained from equations (29) and (30). In the figures illustrated below, the ordinates represent non-dimensional dynamic responses w_{1max}/w_{1st} and w_{2max}/w_{1st}, in which w_{1max} and w_{2max} are the maximum dynamic displacements of the main and the dynamic absorbing shells under a periodic load, and w_{1st} is the static displacement of the main shell under the same load. The abscissas represent the non-dimensional exciting frequency ω_0/ω_{11} in which ω_0 is the frequency of excitation and ω_{11} is the natural circular frequency of the 1st mode of the main shell.

4.1. THE SUPPRESSING EFFECT OF THE SHELL-TYPE DYNAMIC ABSORBER AND THE APPLICABILITY OF THE TUNING METHOD

The shell-type dynamic absorbers with uniformly distributed connecting springs and dampers, and the ones with 6 × 6 sets of concentrated connecting springs and dampers, were designed using the tuning method proposed in this paper, for

TABLE 1

Geometrical and structural constants of the main shell

Width × length ($a \times b$) (cm)	150 × 150
Radius of curvature (R) (cm)	286·25
Mass per unit area (m_1) (Ns/cm)	$2·91 \times 10^{-5}$
Flexural rigidity (D_1) (Ncm)	$7·21 \times 10^3$
Extensional rigidity (K_1)(N/cm)	$8·65 \times 10^4$

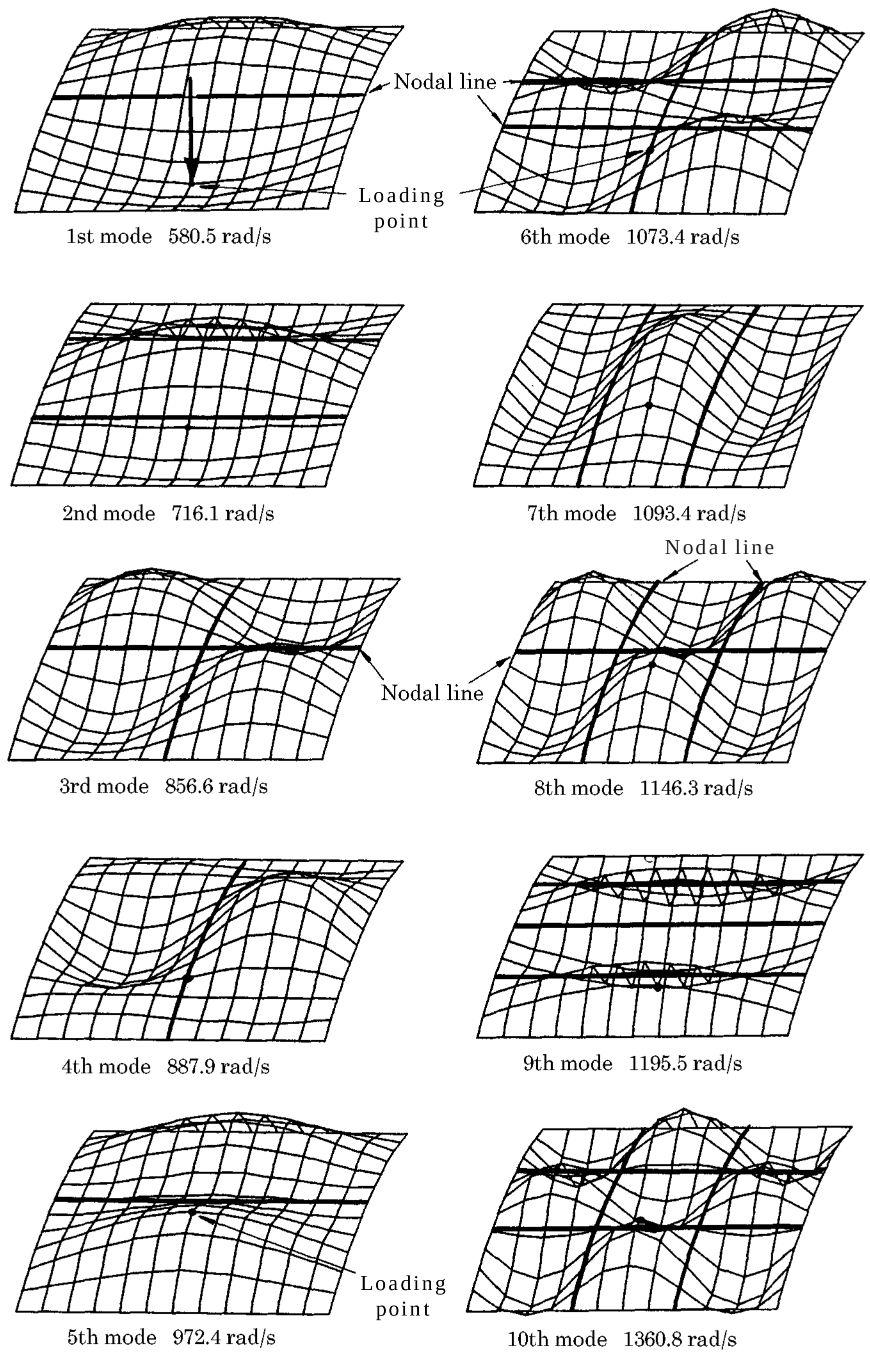

Figure 5. Natural modes and frequencies of the main shell supported simply at the circumference and loading condition.

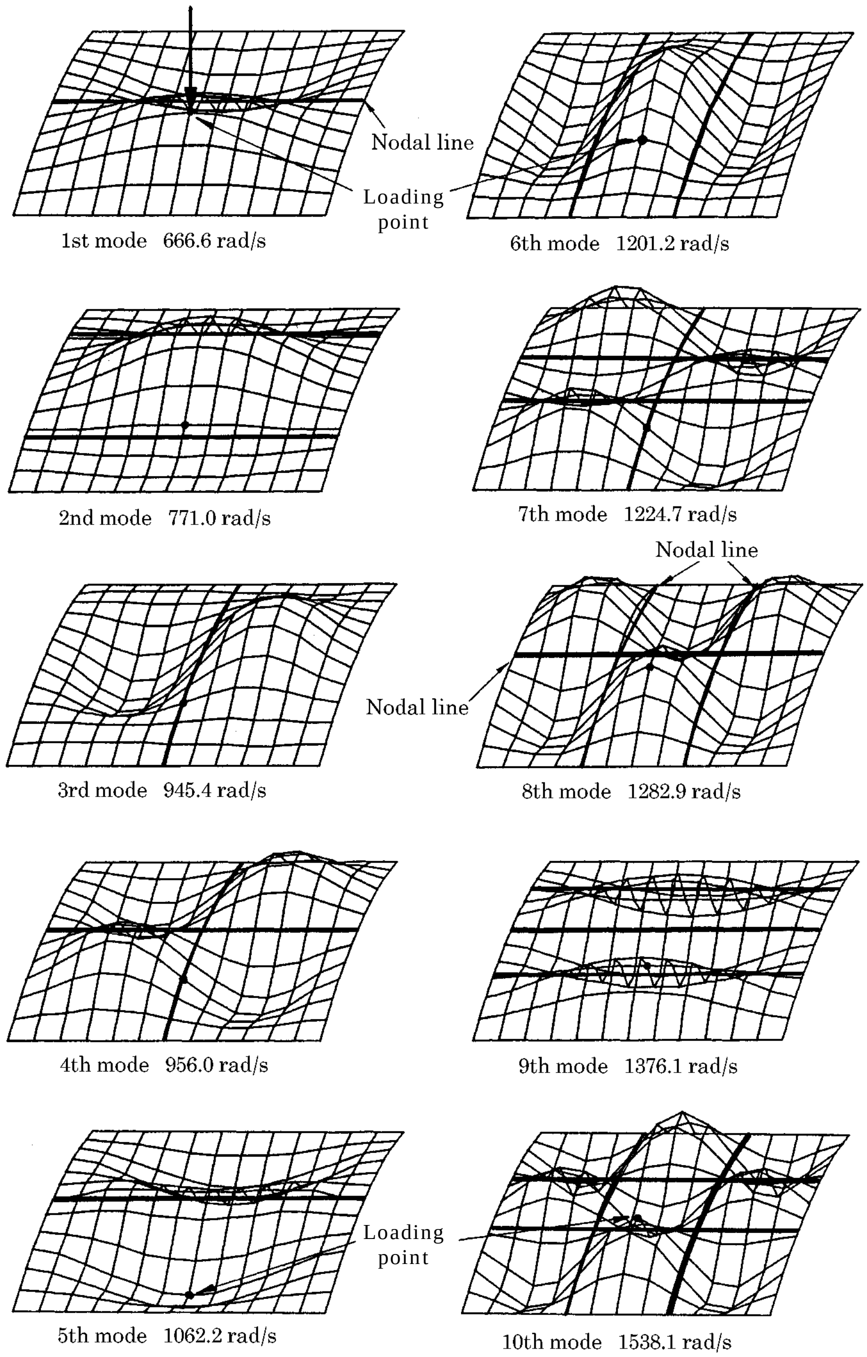

Figure 6. Natural modes and frequencies of the main shell fixed at the circumference and loading condition.

TABLE 2

Geometrical and structural constants of the dynamic absorbing shell

Mass per unit area (m_2) (Ns/cm)	$5{\cdot}52 \times 10^{-6}$
Flexural rigidity (D_2) (Ncm)	$3{\cdot}79 \times 10^{2}$
Extensional rigidity (K_2) (N/cm)	$4{\cdot}55 \times 10^{3}$

control of the 1st mode of the main shell under the condition that the mass ratio $\mu = 0{\cdot}2$ and the magnification factor $Y_{1p} = 4{\cdot}0$. The structural constants of the dynamic absorbing shells are shown in Table 2 and the characteristic constants of the connecting springs and dampers are shown in Table 3. Spring constant $\bar{k}_z$ and damping coefficient $\bar{c}_z$ were estimated from equations (50) and (52) determining the area ($\mathrm{d}x\,\mathrm{d}y$) of each section. This was obtained by dividing the length and the width of the main shell into six equal parts, as is shown in pattern A in Figure 9. Concentrated springs and dampers with the above characteristics were attached to the center of each section.

The resonance curves of the shallow cylindrical main shell and the dynamic absorbing shell, which are interconnected with uniformly distributed connecting springs and dampers, under a harmonic load acting on a loop point of the 1st mode of the main shell (as shown in Figures 5 and 6) are illustrated in Figures 7 and 8. Figure 7 shows the resonance curves of the shell simply supported at the circumference and Figure 8 shows those of the shell fixed at the circumference. Figures 7(b) and 8(b) are the resonance curves of the vertical displacement at a loop point of the 1st mode of the main shell with the shell-type dynamic absorber, and Figures 7(c) and 8(c) are the resonance curves of the vertical displacement at a point on the dynamic absorbing shell, which is just under the loading point of the above main shell. The resonance curves of the main shells without the dynamic absorber are illustrated in Figures 7(a) and 8(a) in order to make the suppressing effect of the shell-type dynamic absorber clear by comparison with the resonance curves in Figures 7(b) and 8(b). In Figures 7 and 8, the resonance curves of the main shell with damping constant $h = 0{\cdot}01$ for each mode of vibration are also illustrated by dotted lines. It is obvious from these that the shell-type dynamic absorber is useful for suppression of vibration of the shallow shell, the tuning

TABLE 3

Spring constants and damping coefficients of the shell-type dynamic absorbers

	Supported simply at circumference	Fixed at circumference
Spring constant (k)* (N/cm^3)	$9{\cdot}83 \times 10^{-2}$	$12{\cdot}96 \times 10^{-2}$
Spring constant (k_z)** (N/cm)	$6{\cdot}14 \times 10^{1}$	$8{\cdot}10 \times 10^{1}$
Damping coefficient (c)* (Ns/cm^3)	$9{\cdot}85 \times 10^{-5}$	$11{\cdot}31 \times 10^{-5}$
Damping coefficient (c_z)** (Ns/cm)	$6{\cdot}16 \times 10^{-2}$	$7{\cdot}07 \times 10^{-2}$

* Uniformly distributed spring and damper.

** Concentrated spring and damper.

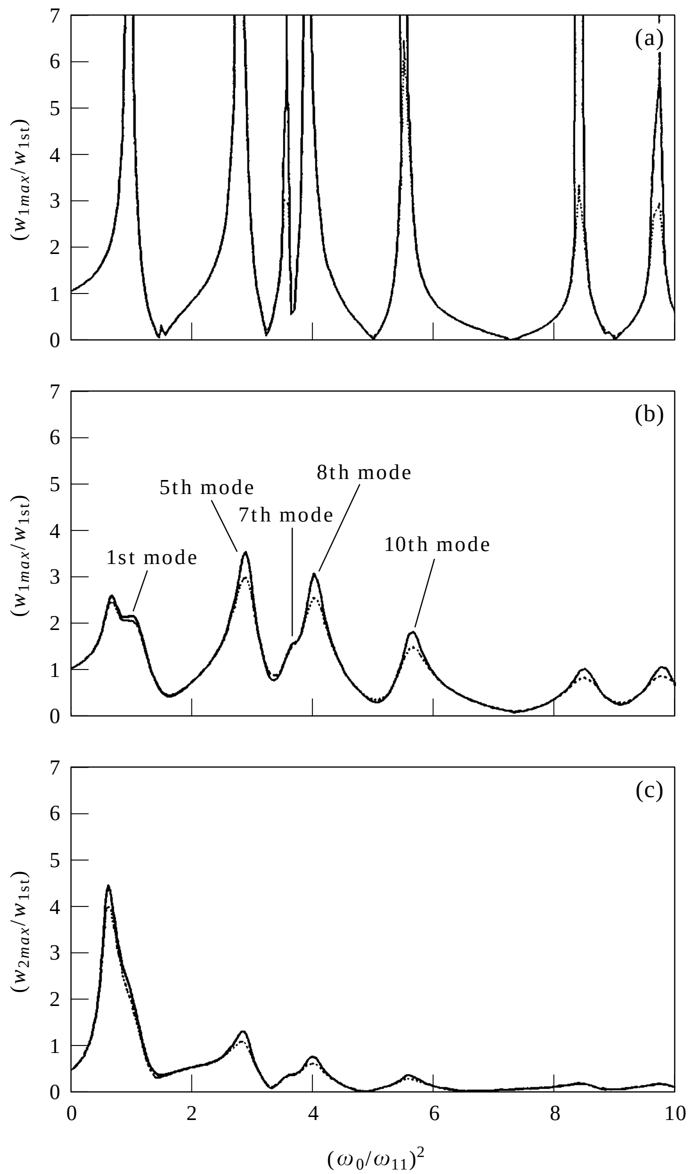

Figure 7. Resonance curves of the main shell and the dynamic absorbing shell supported simply at the circumference: (a) main shell without shell-type dynamic absorber; (b) main shell with shell-type dynamic absorber; (c) dynamic absorbing shell attached to the main shell. ——, Without damping; $\cdots\cdots$, with damping ($h = 0{\cdot}01$).

method presented here is applicable and the responses in the resonance of the main shell are reduced by structural internal damping.

The resonance curves of the main and dynamic absorbing shells which are interconnected with 6×6 sets of concentrated connecting springs and dampers in the vertical direction, corresponding to the resonance curves in Figures 7 and 8, were calculated in order to investigate the applicability of the approximate tuning method. Those resonance curves of the main shells and the dynamic absorbing shells are not illustrated here, because the shapes of the resonance curves of the main shells were almost the same as those in Figures 7(b) and 8(b), and those of the dynamic absorbing shells were the same as those in Figures 7(c) and 8(c). However, the peak values on the resonance curves of the main shell and the dynamic absorbing shell in the vicinity of $\omega_0/\omega_{11} = 1{\cdot}0$ are shown in Table 4. It is evident from Table 4 that the shell-type dynamic absorber with 6×6 sets, or larger sets of concentrated connecting springs and dampers, being tuned by the

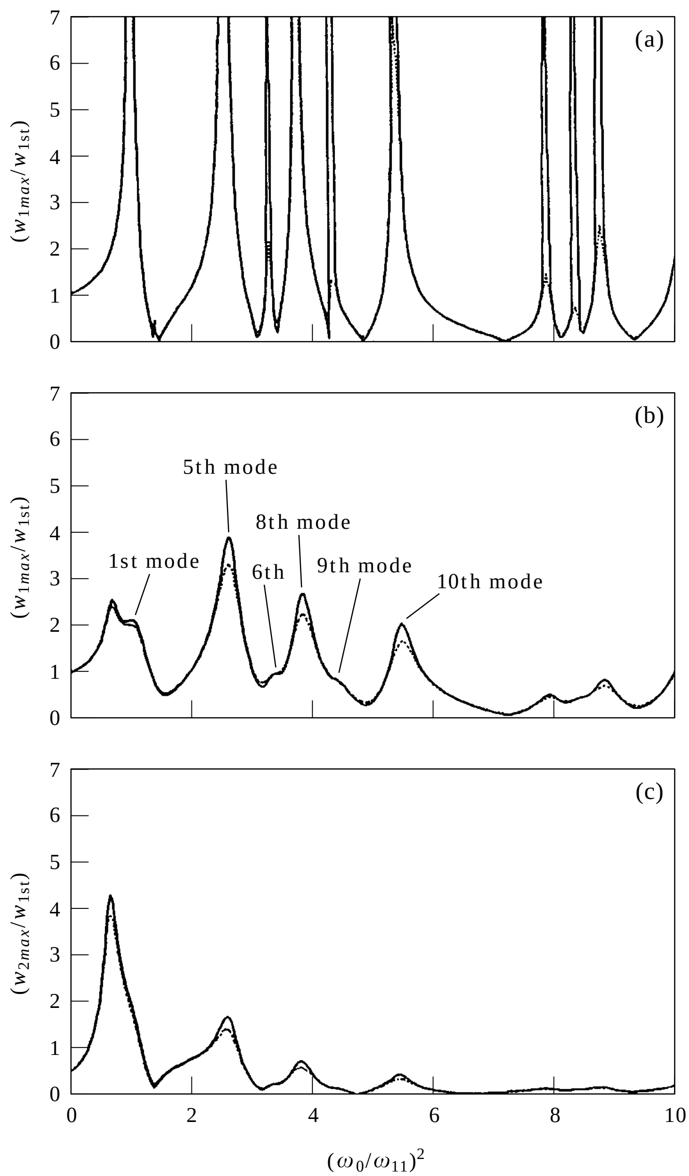

Figure 8. Resonance curves of the main shell and the dynamic absorbing shell fixed at the circumference: (a) main shell without shell-type dynamic absorber; (b) main shell with shell-type dynamic absorber; (c) dynamic absorbing shell attached to the main shell. ——, Without damping; · · · · · ·, with damping ($h = 0{\cdot}01$).

TABLE 4

Peak values of dynamic responses of the main shells and the dynamic absorbing shells in the vicinity of $\omega_0/\omega_{11} = 1{\cdot}0$

	Shell supported simply at the circumference		Shell fixed at the circumference	
Springs and dampers	Main shell	Dynamic absorbing shell	Main shell	Dynamic absorbing shell
Uniformly distributed spring and damper	2·594	4·456	2·541	4·450
Concentrated spring and damper	2·595	4·256	2·530	4·232

approximate tuning method presented here, is applicable for controlling the vibration of the shallow shell. This fact shows that the horizontal connecting springs and dampers do not have a suppressing effect on vibration of the shallow shell with the geometrical and structural constants shown in Table 1.

4.2. THE EFFECT OF VARYING THE ARRANGEMENT OF THE CONCENTRATED CONNECTING SPRINGS AND DAMPERS

The behavior of the suppressing effect on the vibration of the shallow main shell, which was simply supported at the circumference due to variations in the arrangement of the concentrated connecting springs and dampers was also investigated. Five patterns of arrangement of the concentrated springs and dampers, as shown in Figure 9, were considered in this investigation. Pattern A shows a close arrangement of the concentrated connecting springs and dampers, patterns B and C show loose arrangements in the x and y directions, and patterns D and E show loose arrangements at the circumference where the dynamic displacement is small.

The resonance curves of the shallow shells with the shell-type dynamic vibration absorber with concentrated connecting springs and dampers arranged in patterns mentioned above are illustrated in Figure 10. The resonance curves are response curves of the vertical displacements at a loop point of the 1st mode of the main shell under a harmonic load acting on the same point (as shown Figure 5). Figure 10(a) shows the resonance curve of the main shell with a shell-type dynamic absorber with uniformly distributed connecting springs and dampers. Figures 10(b)–(f) show those of the main shell with the shell-type dynamic absorber, having concentrated connecting springs and dampers with five patterns of arrangement, as shown in Figure 9. The resonance curves of the main shell with damping constant $h = 0{\cdot}01$ for each mode of vibration are also illustrated by dotted lines.

The shell-type dynamic absorber with connecting springs and dampers with the arrangement of pattern A has the same suppressing effect as the one with uniformly distributed connecting springs and dampers, as shown in Figure 10(a) and (b). The change of the peak in the vicinity of the 1st natural circular frequency

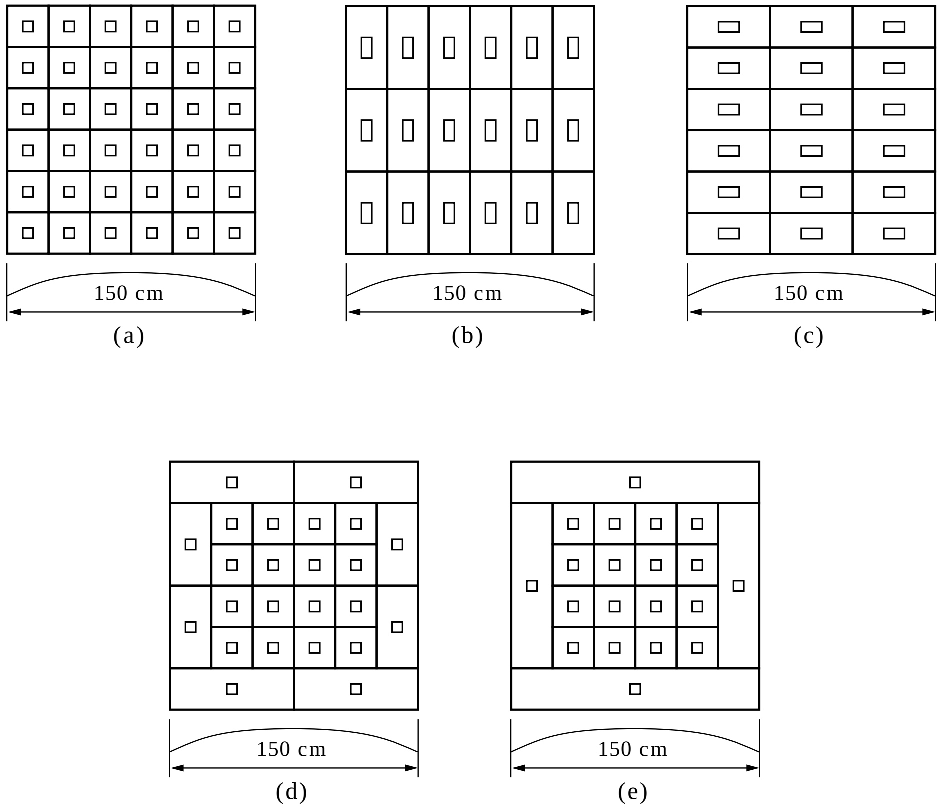

Figure 9. Patterns of arrangement of the concentrated connecting springs and dampers in the vertical direction. (a) Pattern A; (b) pattern B; (c) pattern C; (d) pattern D; (e) pattern E.

is shown in Table 4. The shell-type dynamic absorber, with connecting springs and dampers with the arrangement of pattern B, has no effect on the suppression for the 5th mode because there are not many connecting springs and dampers arranged on the loop line (parallel to the nodal line) of the 5th mode. However, the suppressing effects on the 7th, 8th and 10th modes show, because there are many connecting springs and dampers on the loop line of those modes. The shell-type dynamic absorber, with connecting springs and dampers with the arrangement of pattern C, has no effect on the 1st and 7th modes because the concentrated springs and dampers are arranged in the position (line) away from the loop line of those modes. The suppressing effect of the shell-type dynamic absorber with connecting springs and dampers with the arrangement of pattern D on the 1st and 8th modes are better than those of pattern E, because concentrated springs and dampers at the circumference of the arrangement of pattern D are on the loops of the 1st and 8th modes.

From the above numerical examples, the following facts were revealed: (1) the shell-type dynamic absorber with 6×6 sets of concentrated connecting springs and dampers in the vertical direction has the same suppressing effect as that with uniformly distributed connecting springs and dampers; and (2) the suppressing effect on vibration decreases when the number of sets of the concentrated

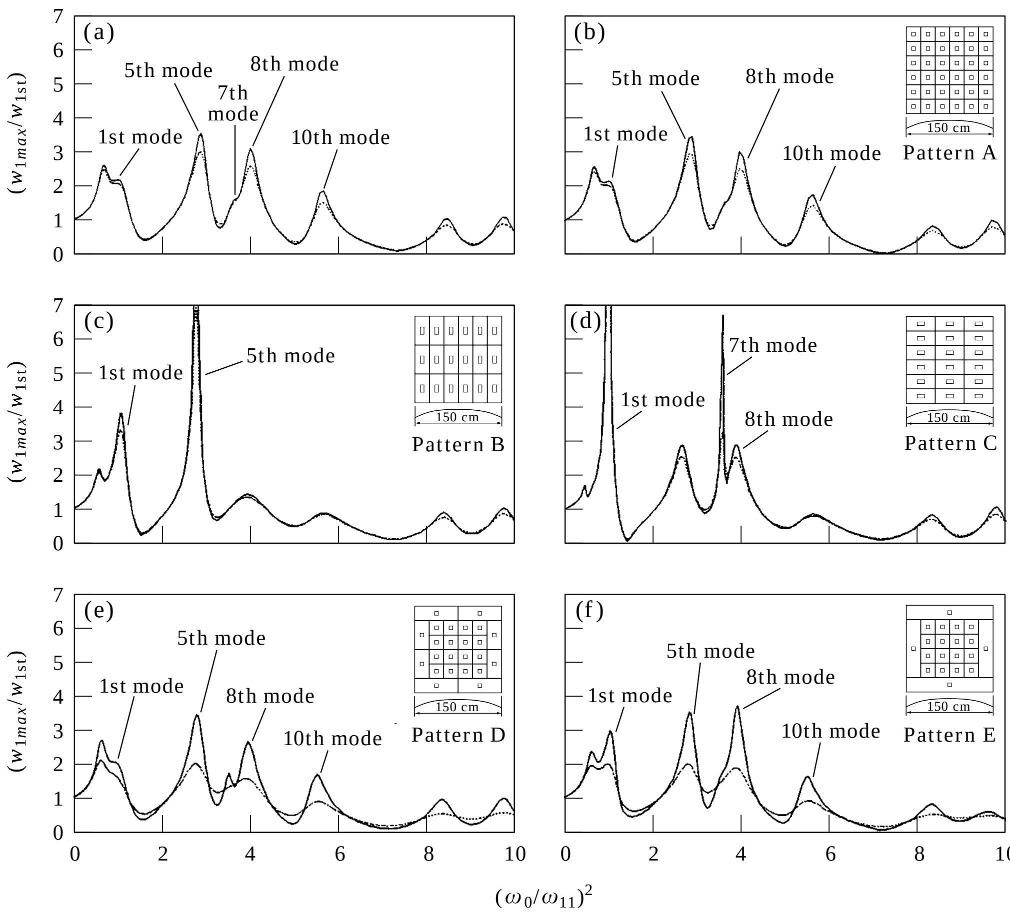

Figure 10. Resonance curves of the main shell with shell-type dynamic absorber: (a) uniformly distributed connecting springs and dampers; (b) concentrated connecting springs and dampers with pattern A; (c) concentrated connecting springs and dampers with pattern B; (d) concentrated connecting springs and dampers with pattern C; (e) concentrated connecting springs and dampers with pattern D; (f) concentrated connecting springs and dampers with pattern E. ——, Without damping; · · · · · · , with damping ($h = 0{\cdot}01$).

connecting springs and dampers (the number of divisions of the shell) decreases, and when the concentrated springs and dampers are arranged away from the loops of the modes.

5. CONCLUDING REMARKS

A shell-type dynamic vibration absorber for controlling the vibration of the shallow shell (main shell) under a harmonic load was proposed, which consists of a shallow shell (dynamic absorbing shell) under the same boundary condition as the main shell, and has connecting springs and dampers between both shells. For practical use, the connecting springs and dampers were arranged in a concentrated fashion and an approximate tuning method for this new dynamic vibration absorber was also developed.

The equation of motion of the main shell with the shell-type dynamic absorber with uniformly distributed connecting springs and dampers, rearranged using the modal co-ordinates of the main shell, was shown to be identical to that of the two-degree-of-freedom system (TDOF system) with two masses and three springs. The equation of motion of the shell with a shell-type dynamic absorber with concentrated connecting springs and dampers in the vertical direction was also shown to be similar to that of the TDOF system. An optimum tuning method for the shell-type dynamic absorber with uniformly distributed connecting springs and dampers was presented by applying the optimum tuning conditions for a dynamic absorber (subsystem) in a TDOF system with two masses and three springs, obtained by the Den Hartog method. Subsequently, an approximate tuning method for the shell-type dynamic absorber with concentrated connecting springs and dampers was also presented.

From numerical investigation, the following facts were obtained: (1) for suppression of vibration of the shallow shell with and without structural internal damping, the shell-type dynamic vibration absorber with uniformly distributed connecting springs and dampers is useful and the tuning method presented here is applicable; (2) the shell-type dynamic absorbers with 6×6 sets or more sets of concentrated connecting springs and dampers in the vertical direction are useful, as well as is the shell-type dynamic absorber with uniformly distributed connecting springs and dampers. The approximate tuning method proposed here is also applicable; (3) the accuracy of the approximate tuning method is improved when the number of sets of concentrated connecting springs and dampers is increased and the concentrated connecting springs and dampers are arranged closely in loop positions where the dynamic displacement is large.

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